ANALYSIS OF THE AXISYMMETRICAL IONIZED GAS BOUNDARY LAYER ADJACENT TO POROUS CONTOUR OF THE BODY OF REVOLUTION

by

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The ionized gas flow in the boundary layer on bodies of revolution with porous contour is studied in this paper. The gas electroconductivity is assumed to be a function of the longitudinal co-ordinate, x. The problem is solved using Saljnikov's version of the general similarity method. This paper is an extension of Saljnikov's generalized solutions and their application to a particular case of magnetohydrodynamic flow. Generalized boundary layer equations have been numerically solved in a four-parametric localized approximation and characteristics of some physical quantities in the boundary layer has been studied.

Key words: *boundary layer, ionized gas, body of revolution, porous contour, magnetic field*

Introduction

This paper is a continuation of investigations of the planar flow in the dissociated and ionized gas boundary layer. The ionized gas flows in the conditions of equilibrium ionization. The contour of the body within the fluid is porous.

The general similarity method was first introduced by Loitsianskii [1, 2] and later improved by Saljnikov [3], and Saljnikov and Dallmann [4]. Both versions are based on momentum equations and corresponding sets of similarity parameters. Loitsianskii's method was applied to problems of the dissociated gas flow in the boundary layer [5, 6]. Saljnikov's version was used for the temperature boundary layer [7, 8], the MHD boundary layer theory [9-11], and for solution of the dissociated and ionized gas flow in the boundary layer [12-16]. In this paper, Saljnikov's version of the general similarity method has been applied.

Mathematical model

At supersonic speeds of aircrafts through the Earth's atmosphere, the temperature in the viscous boundary layer increases significantly. Gas dissociation and then ionization occur and the air becomes a multicomponent mixture of atoms, electrons and positively charged ions of oxygen, nitrogen, *etc.* [17-19]. When the temperature in the air flow is high enough, the thermo-chemical equilibrium is established. The electroconductivity, σ, is an important property of the ionized gas and it is a function of the temperature *i. e.* enthalpy [20].

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When the ionized gas flows in the magnetic field of the intensity $B_m = B_m(x)$, an electric current is formed. Lorentz force and Joule's heat are the consequence of mutual effects of the magnetic field, fluid velocity and electric current density [20]. The electroconductivity is also assumed to be a function of the longitudinal co-ordinate x, thus the electroconductivity variation law can be written:

$$\sigma = \sigma(x) \tag{1}$$

Hence, in the case of the ionized gas flow in the magnetic field under conditions of equilibrium ionization, the equations of steady laminar boundary layer on bodies of revolution with porous wall [12, 20, 21] take the form:

$$\frac{\partial}{\partial x}(\rho u r^j) + \frac{\partial}{\partial y}(\rho v r^j) = 0, \quad (j = 1) \tag{2}$$

as the continuity equation:

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \rho_e u_e \frac{\mathrm{d}u_e}{\mathrm{d}x} + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) + \sigma B_m^2 (u_e - u) \tag{3}$$

as the dynamic equation, and:

$$\rho u \frac{\partial h}{\partial x} + \rho v \frac{\partial h}{\partial y} = -u \rho_e u_e \frac{\mathrm{d}u_e}{\mathrm{d}x} + \mu \left(\frac{\partial u}{\partial y}\right)^2 + \frac{\partial}{\partial y}\left(\frac{\mu}{\mathrm{Pr}} \frac{\partial h}{\partial y}\right) + \sigma B_m^2 (u^2 - u u_e) \tag{4}$$

as the energy equation.

The boundary conditions are:

$$\begin{aligned} u = 0, \quad v = v_w(x), \quad h = h_w = \text{const.} \quad \text{for} \quad y = 0, \\ u \to u_e(x), \quad h \to h_e(x) \quad \text{for} \quad y \to \infty. \end{aligned} \tag{5}$$

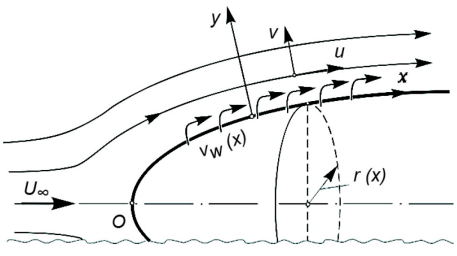

Figure 1. Ionized gas flow adjacent to the body of revolution

Lorentz force and Joule's heat are determined respectively for the terms $\sigma B_m^2 (u_e - u)$ and $\sigma B_m^2 (u^2 - u u_e)$ [20]. The following symbols are used: x, y – the longitudinal and transversal co-ordinates, u (x, y) – the longitudinal velocity projection in the boundary layer, v (x, y) – the transversal velocity projection, ρ – the density, h – the enthalpy, μ – the dynamic viscosity, r (x) – the radius of the body of revolution in the meridian plane (fig. 1), and $\mathrm{Pr} = \mu c_p/\lambda$. Here, λ denotes the thermal electroconductivity coefficient and c_p the specific heat of the ionized gas at constant pressure. The subscript e stands for physical quantities at the outer edge of the boundary layer ($y \to \infty$) and the subscript w denotes the values on the wall of the body of revolution ($y = 0$). The velocity $v_w(x)$ at which the gas flows perpendicularly through the porous wall of the body of revolution can be positive (at injection) or negative (at suction).

The continuity eq. (2) can be written in a more convenient form:

$$\frac{\partial}{\partial x}\left[\rho u\left(\frac{r}{L}\right)^{j}\right]+\frac{\partial}{\partial y}\left[\rho v\left(\frac{r}{L}\right)^{j}\right]=0, \qquad (L=\text{const.},\ \ j=1) \tag{6}$$

where L is the constant length whose value can be equal to one [6].

Transformation of the equations

In order to apply the general similarity method, analogous to already solved problems of axisymmetrical compressible fluid flow in the boundary layer [6, 22], new variables are introduced:

$$s(x)=\frac{1}{\rho_0\mu_0}\int_0^x \rho_w\mu_w\left(\frac{r}{L}\right)^{2j}\mathrm{d}x, \qquad z(x,y)=\left(\frac{r}{L}\right)^{j}\int_0^y \frac{\rho}{\rho_0}\mathrm{d}y, \qquad (j=1) \tag{7}$$

Here, ρ_0, μ_0 and $\rho_w(x)$, $\mu_w(x)$ denote the known values of the density and the dynamic viscosity of the gas at some point of the boundary layer (subscript 0) and on the wall of the body of revolution (subscript w).

The stream function $\psi(s,z)$ is introduced using the relations:

$$u=\frac{\partial\psi}{\partial z}, \qquad \tilde{v}=\frac{1}{\left(\frac{r}{L}\right)^{2j}}\frac{\rho_0\mu_0}{\rho_w\mu_w}\left[u\frac{\partial z}{\partial x}+v\frac{\rho}{\rho_0}\left(\frac{r}{L}\right)^{j}\right]=-\frac{\partial\psi}{\partial s}, \qquad (j=1) \tag{8}$$

that follow from the continuity eq. (6).

Since the boundary condition for the velocity at the inner edge of the boundary layer (5) does not equal zero, $v = v_w(x) \neq 0$, as with incompressible fluid [2], the stream function $\psi(s,z)$ is divided into two parts:

$$\psi(s,z)=\psi_w(s)+\bar{\psi}(s,z), \qquad \bar{\psi}(s,0)=0 \tag{9}$$

Here, $\psi_w(s) = \psi(s,0)$ stands for the stream function of the flow adjacent to the wall ($z = 0$) for the body of revolution.

Further on, following variables are introduced:

$$\begin{gathered}
s=s, \qquad \eta(s,z)=\frac{u_e^{b/2}}{K(s)}z=\frac{B(s)}{\Delta^{**}(s)}\,z,\\
\bar{\psi}(s,z)=u_e^{1-b/2}K(s)\Phi(s,\eta)=\frac{u_e\Delta^{**}(s)}{B(s)}\Phi(s,\eta),\\
h(s,z)=h_1\bar{h}(s,\eta), \qquad h_1=\text{const.}, \qquad K(s)=\left(a\nu_0\int_0^s u_e^{b-1}\mathrm{d}s\right)^{1/2},\\
B(s)=\int_0^\infty\frac{\partial\Phi}{\partial\eta}\left(1-\frac{\partial\Phi}{\partial\eta}\right)\mathrm{d}\eta, \qquad a,b=\text{const.}
\end{gathered} \tag{10}$$

where ν_0 is the kinematic viscosity at a concrete point.

The following symbols are used: $\eta(s, z)$ – the newly introduced transversal variable, $\Phi(s, \eta)$ – the non-dimensional stream function, $\bar{h}$ – the non-dimensional enthalpy, h_1 – the stagnation enthalpy in the outer flow, and a, b – arbitrary constants. In this paper the subscript 1 is used for h_1 as usually used in the literature [5, 6, 12] that deals with the dissociated and ionized gas boundary layer flow.

Using (7)-(10), the eqs. (3) and (4) and boundary conditions (5) are transformed into the following system with the corresponding boundary conditions:

$$\frac{\partial}{\partial\eta}\left(Q\frac{\partial^2\Phi}{\partial\eta^2}\right)+\frac{aB^2+(2-b)f}{2B^2}\Phi\frac{\partial^2\Phi}{\partial\eta^2}+\frac{f}{B^2}\left[\frac{\rho_e}{\rho}-\left(\frac{\partial\Phi}{\partial\eta}\right)^2\right]+\frac{g}{B^2}\frac{\rho_e}{\rho}\left(1-\frac{\partial\Phi}{\partial\eta}\right)+$$

$$+\frac{\Lambda}{B}\frac{\partial^2\Phi}{\partial\eta^2}=\frac{u_e}{u_e'}\frac{f}{B^2}\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial^2\Phi}{\partial s\partial\eta}-\frac{\partial\Phi}{\partial s}\frac{\partial^2\Phi}{\partial\eta^2}\right),$$

$$\frac{\partial}{\partial\eta}\left(\frac{Q}{\mathrm{Pr}}\frac{\partial\bar{h}}{\partial\eta}\right)+\frac{aB^2+(2-b)f}{2B^2}\Phi\frac{\partial\bar{h}}{\partial\eta}-\frac{2\kappa f}{B^2}\frac{\rho_e}{\rho}\frac{\partial\Phi}{\partial\eta}+2\kappa Q\left(\frac{\partial^2\Phi}{\partial\eta^2}\right)^2+\frac{\Lambda}{B}\frac{\partial\bar{h}}{\partial\eta}+$$

$$+\frac{2\kappa g}{B^2}\frac{\rho_e}{\rho}\frac{\partial\Phi}{\partial\eta}\left(\frac{\partial\Phi}{\partial\eta}-1\right)=\frac{u_e}{u_e'}\frac{f}{B^2}\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial\bar{h}}{\partial s}-\frac{\partial\Phi}{\partial s}\frac{\partial\bar{h}}{\partial\eta}\right) \tag{11}$$

$$\Phi(s,\eta)=0,\qquad \frac{\partial\Phi}{\partial\eta}=0,\qquad \bar{h}=\bar{h}_w\qquad \text{for}\qquad \eta=0,$$

$$\frac{\partial\Phi}{\partial\eta}\to 1,\qquad \bar{h}\to\bar{h}_e(s)=1-\kappa\qquad \text{for}\qquad \eta\to\infty$$

where prim (′) stands for a derivative per the variable s.

The usual quantities in the boundary layer theory [5, 12] are introduced to the system (11): the conditional displacement thickness $\Delta^*(s)$, the conditional momentum loss thickness $\Delta^{**}(s)$, the conditional thickness $\Delta_1^*(s)$, the non-dimensional friction function $\zeta(s)$, and the characteristic boundary layer function F_{mp}. They are defined:

$$\Delta^*(s)=\int_0^\infty\left(\frac{\rho_e}{\rho}-\frac{u}{u_e}\right)\mathrm{d}z,\qquad \Delta^{**}(s)=\int_0^\infty\frac{u}{u_e}\left(1-\frac{u}{u_e}\right)\mathrm{d}z,\qquad \Delta_1^*(s)=\int_0^\infty\frac{\rho_e}{\rho}\left(1-\frac{u}{u_e}\right)\mathrm{d}z$$

$$H=\frac{\Delta^*}{\Delta^{**}},\qquad H_1=\frac{\Delta_1^*}{\Delta^{**}},\qquad \zeta(s)=\left[\frac{\partial\dfrac{u}{u_e}}{\partial\dfrac{z}{\Delta^{**}}}\right]_{z=0}=B\left(\frac{\partial^2\Phi}{\partial\eta^2}\right)_{\eta=0},\qquad Z^{**}=\frac{\Delta^{**2}}{\nu_0} \tag{12}$$

$$\frac{dZ^{**}}{ds}=\frac{F_{\mathrm{mp}}}{u_e},\qquad F_{\mathrm{mp}}=2[\zeta-(2+H)f]-2gH_1-2\Lambda$$

While derivating the momentum equation, the expressions for $\Delta^*(s)$, $\Delta^{**}(s)$, and $\Delta_1^*(s)$ were obtained. The first two of these expressions have the same form as the corre-

sponding expressions defining the displacement thickness and momentum loss thickness of the incompressible fluid. Since these expressions have the same form, these values are referred to as conditional thicknesses both in this paper and in the literature [5, 12].

In eq. (11), the form parameter, $f(s)$, the magnetic parameter, $g(s)$, the porosity parameter, $\Lambda(s)$, and the local compressibility parameter, $\kappa(s)$, [5] are all basic parameters that depend on the conditions at the outer or inner edge of the boundary layer [22]. They are defined:

$$f(s)=\frac{u_{\mathrm{e}}'\Delta^{**2}}{\nu_0}=u_{\mathrm{e}}'Z^{**}=f_1(s),\qquad g(s)=SZ^{**}=g_1(s),\qquad S=\frac{1}{\left(\frac{r}{L}\right)^{2j}}\frac{\rho_0\mu_0}{\rho_w\mu_w}\frac{\sigma B_{\mathrm{m}}^2}{\rho_{\mathrm{e}}},$$
$$\Lambda(s)=-\frac{1}{\left(\frac{r}{L}\right)^{j}}\frac{\mu_0}{\mu_w}v_w\frac{\Delta^{**}}{\nu_0}=-\frac{V_{\mathrm{w}}\Delta^{**}}{\nu_0}=\Lambda_1(s),\qquad V_w=\frac{1}{\left(\frac{r}{L}\right)^{j}}\frac{\mu_0}{\mu_w}v_w \tag{13}$$

where $V_{\mathrm{w}}(s)$ denotes the conditional transversal velocity at the inner edge of the boundary layer. The local compressibility parameter is determined:

$$\kappa=f_0(s)=\frac{u_{\mathrm{e}}^2}{2h_1} \tag{14}$$

The non-dimensional function $Q=\rho\mu/(\rho_{\mathrm{w}}\mu_{\mathrm{w}})$ and the density ratio ρ_{e}/ρ that appear in the system (11) depend on the thermo-dynamical properties of the ionized gas. Analytical expressions of these functions are needed for numerical solution of the system (11).

Note that both equations of the system (11) contain $u_{\mathrm{e}}/u_{\mathrm{e}}'$ in their terms. Consequently, the solution of the system will depend on a particular form of the law of the given velocity $u_{\mathrm{e}}(s)$ at the outer edge of the boundary layer. As a result, the obtained system is not generalized in terms of Loitsianskii [2]. The analysis has shown that it is not possible to derive generalized boundary layer equations for the studied problem using the functions $\Phi(s,\eta)$ and $\bar{h}(s,\eta)$.

Generalized boundary layer equations

In order to bring the governing equation system into a generalized form, a new stream function, Φ, and a non-dimensional enthalpy, $\bar{h}$, are introduced:

$$\bar{\psi}(s,\eta)=\frac{u_{\mathrm{e}}(s)\Delta^{**}(s)}{B(s)}\Phi[(\eta,\kappa,(f_k),(g_k),(\Lambda_k)],$$
$$h(s,\eta)=h_1\bar{h}[(\eta,\kappa,(f_k),(g_k),(\Lambda_k)] \tag{15}$$

In eq. (15), (f_k) denotes a set of form parameters of Loitsianskii's type [2], (g_k) stands for a set of magnetic parameters, and (Λ_k) denotes a set of porosity parameters of the porous wall [21]. These are new independent variables (instead of the variable s) and they are defined:

$$f_k(s)=u_{\mathrm{e}}^{k-1}u_{\mathrm{e}}^{(k)}Z^{**^k},\quad g_k(s)=u_{\mathrm{e}}^{k-1}S^{(k-1)}Z^{**^k},\quad \Lambda_k(s)=-u_{\mathrm{e}}^{k-1}\left(\frac{V_w}{\sqrt{\nu_0}}\right)^{(k-1)}Z^{**^{k-1/2}},\quad (k=1,\,2,\,3,\,...) \tag{16}$$

Each set of parameters (16) satisfies a corresponding recurrent simple differential equation:

$$\frac{u_e}{u_{\mathrm{e}}'}f_1\frac{\mathrm{d}\kappa}{\mathrm{d}s}=2\kappa f_1=\theta_0,\qquad \frac{u_e}{u_e'}f_1\frac{\mathrm{d}f_k}{\mathrm{d}s}=[(k-1)f_1+kF_{mp}]f_k+f_{k+1}=\theta_k$$

$$\frac{u_e}{u_{\mathrm{e}}'}f_1\frac{\mathrm{d}g_k}{\mathrm{d}s}=[(k-1)f_1+kF_{mp}]g_k+g_{k+1}=\gamma_k \tag{17}$$

$$\frac{u_e}{u_{\mathrm{e}}'}f_1\frac{\mathrm{d}\Lambda_k}{\mathrm{d}s}=\left[(k-1)f_1+\frac{2k-1}{2}F_{mp}\right]\Lambda_k+\Lambda_{k+1}=\chi_k,\qquad (k=1,\,2,\,3,\,...)$$

The first parameters in the introduced sets (k = 1) represent the already defined form parameter $f_1=f=u_{\mathrm{e}}'Z^{**}$, magnetic parameter $g_1=g=SZ^{**}$, and porosity parameter $\Lambda_1=\Lambda=-[V_{\mathrm{w}}/(\nu_0)^{1/2}]Z^{**1/2}=-V_{\mathrm{w}}\Delta^{**}/\nu_0$ (13).

Applying similarity transformations (15), a generalized boundary layer equation system of ionized gas flow on bodies of revolution with porous wall has been obtained:

$$\frac{\partial}{\partial\eta}\left(Q\frac{\partial^2\Phi}{\partial\eta^2}\right)+\frac{aB^2+(2-b)f_1}{2B^2}\Phi\frac{\partial^2\Phi}{\partial\eta^2}+\frac{f_1}{B^2}\left[\frac{\rho_e}{\rho}-\left(\frac{\partial\Phi}{\partial\eta}\right)^2\right]+\frac{g_1}{B^2}\frac{\rho_e}{\rho}\left(1-\frac{\partial\Phi}{\partial\eta}\right)+\frac{\Lambda_1}{B}\frac{\partial^2\Phi}{\partial\eta^2}=$$

$$=\frac{1}{B^2}\left[\sum_{k=0}^{\infty}\theta_k\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial^2\Phi}{\partial\eta\partial f_k}-\frac{\partial\Phi}{\partial f_k}\frac{\partial^2\Phi}{\partial\eta^2}\right)+\sum_{k=1}^{\infty}\gamma_k\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial^2\Phi}{\partial\eta\partial g_k}-\frac{\partial\Phi}{\partial g_k}\frac{\partial^2\Phi}{\partial\eta^2}\right)+\right.$$

$$\left.+\sum_{k=1}^{\infty}\chi_k\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial^2\Phi}{\partial\eta\partial\Lambda_k}-\frac{\partial\Phi}{\partial\Lambda_k}\frac{\partial^2\Phi}{\partial\eta^2}\right)\right]$$

$$\frac{\partial}{\partial\eta}\left(\frac{Q}{\mathrm{Pr}}\frac{\partial\bar{h}}{\partial\eta}\right)+\frac{aB^2+(2-b)f_1}{2B^2}\Phi\frac{\partial\bar{h}}{\partial\eta}-\frac{2\kappa f_1}{B^2}\frac{\rho_{\mathrm{e}}}{\rho}\frac{\partial\Phi}{\partial\eta}+2\kappa Q\left(\frac{\partial^2\Phi}{\partial\eta^2}\right)^2- \tag{18}$$

$$-\frac{2\kappa g_1}{B^2}\frac{\rho_{\mathrm{e}}}{\rho}\frac{\partial\Phi}{\partial\eta}\left(1-\frac{\partial\Phi}{\partial\eta}\right)+\frac{\Lambda_1}{B}\frac{\partial\bar{h}}{\partial\eta}=\frac{1}{B^2}\left[\sum_{k=0}^{\infty}\theta_k\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial\bar{h}}{\partial f_k}-\frac{\partial\Phi}{\partial f_k}\frac{\partial\bar{h}}{\partial\eta}\right)+\right.$$

$$\left.+\sum_{k=1}^{\infty}\gamma_k\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial\bar{h}}{\partial g_k}-\frac{\partial\Phi}{\partial g_k}\frac{\partial\bar{h}}{\partial\eta}\right)+\sum_{k=1}^{\infty}\chi_k\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial\bar{h}}{\partial\Lambda_k}-\frac{\partial\Phi}{\partial\Lambda_k}\frac{\partial\bar{h}}{\partial\eta}\right)\right];$$

$$\Phi=0,\qquad \frac{\partial\Phi}{\partial\eta}=0,\qquad \bar{h}=\bar{h}_w\qquad \text{for}\qquad \eta=0,$$

$$\frac{\partial\Phi}{\partial\eta}\to 1,\qquad \bar{h}\to\bar{h}_e(s)=1-\kappa\qquad \text{for}\qquad \eta\to\infty$$

Here, distribution of the velocity $u_{\mathrm{e}}(s)$ at the outer edge of the boundary layer does not figure explicitly, so the system is generalized. In the case of a non-porous wall of the body

within the fluid ($v_w = 0$), all the porosity parameters equal zero and eq. (18) comes down to the system for the flow adjacent to a non-porous wall of the body of revolution [23]. Note that for $j = 0$, the system (18) is absolutely identical to the corresponding system for the case of planar ionized gas flow [24].

Since a numerical solution is practically impossible, the obtained equation system is solved in an n-parametric localized approximation (solution for a relatively small number of parameters). In the four parametric ($\kappa = f_0 \neq 0$, $f_1 = f \neq 0$, $g_1 = g \neq 0$, $\Lambda_1 = \Lambda \neq 0$; $f_k = g_k = \Lambda_k = 0$ for $k \geq 2$) three times localized approximation ($\partial/\partial\kappa = 0$, $\partial/\partial g_1 = 0$, $\partial/\partial\Lambda_1 = 0$) the obtained equation system (18) is significantly simplified. Thus it comes down to the equation system:

$$\frac{\partial}{\partial\eta}\left(Q\frac{\partial^2\Phi}{\partial\eta^2}\right)+\frac{aB^2+(2-b)f}{2B^2}\Phi\frac{\partial^2\Phi}{\partial\eta^2}+\frac{f}{B^2}\left[\frac{\rho_e}{\rho}-\left(\frac{\partial\Phi}{\partial\eta}\right)^2\right]+\frac{g}{B^2}\frac{\rho_e}{\rho}\left(1-\frac{\partial\Phi}{\partial\eta}\right)+$$
$$+\frac{\Lambda}{B}\frac{\partial^2\Phi}{\partial\eta^2}=\frac{F_{mp}f}{B^2}\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial^2\Phi}{\partial\eta\partial f}-\frac{\partial\Phi}{\partial f}\frac{\partial^2\Phi}{\partial\eta^2}\right)$$
$$\frac{\partial}{\partial\eta}\left(\frac{Q}{\mathrm{Pr}}\frac{\partial\bar{h}}{\partial\eta}\right)+\frac{aB^2+(2-b)f}{2B^2}\Phi\frac{\partial\bar{h}}{\partial\eta}-\frac{2\kappa f}{B^2}\frac{\rho_e}{\rho}\frac{\partial\Phi}{\partial\eta}+2\kappa Q\left(\frac{\partial^2\Phi}{\partial\eta^2}\right)^2-$$
$$-\frac{2\kappa g}{B^2}\frac{\rho_e}{\rho}\left(1-\frac{\partial\Phi}{\partial\eta}\right)\frac{\partial\Phi}{\partial\eta}+\frac{\Lambda}{B}\frac{\partial\bar{h}}{\partial\eta}=\frac{F_{mp}f}{B^2}\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial\bar{h}}{\partial f}-\frac{\partial\Phi}{\partial f}\frac{\partial\bar{h}}{\partial\eta}\right) \tag{19}$$
$$\Phi=0,\qquad \frac{\partial\Phi}{\partial\eta}=0,\qquad \bar{h}=\bar{h}_w=\text{const.}\qquad \text{for}\qquad \eta=0,$$
$$\frac{\partial\Phi}{\partial\eta}\to 1,\qquad \bar{h}\to\bar{h}_e(s)=1-\kappa\qquad \text{for}\qquad \eta\to\infty$$
$$\Phi=\Phi^1=\Phi^1(\eta,\kappa,f,g,\Lambda),\quad \bar{h}=\bar{h}^1=\bar{h}^1(\eta,\kappa,f,g,\Lambda)$$

in which the subscript 1 is left out in the first parameters and where the characteristic function F_{mp} is determined by the relation eq. (12).

The system eq. (19) presents the general mathematical model of the ionized gas flow in the boundary layer adjacent to the porous wall on bodies of revolution. Due to the performed localization, the parameters $\kappa = f_0$, g, and Λ become simple parameters. Hence, the system (19) is solved for in advance given values of these parameters.

Numerical solution

In eq. (19), analogous to the dissociated air [5], approximate dependences are adopted for the function Q and the density ratio ρ_e/ρ:

$$Q=\frac{\rho\mu}{\rho_w\mu_w}=Q(\bar{h})\approx\left(\frac{\bar{h}_w}{\bar{h}}\right)^{1/3},\qquad \frac{\rho_e}{\rho}\approx\frac{\bar{h}}{1-\kappa} \tag{20}$$

Correct laws on distributions of these quantities can be obtained only by detailed analysis and using thermodynamic tables for ionized air, but this is not a priority of our study. Since, Prandtl number slightly depends on the temperature [5], the equations of the system (19) are solved for the constant value Pr = 0.712. For constants a and b, the usual values are adopted [4]: $a = 0.4408$ and $b = 5.7140$.

The system of differential partial equations of the third order (19) is numerically solved after the order of the dynamic equation has been decreased by the change:

$$\frac{u}{u_e} = \frac{\partial \Phi}{\partial \eta} = \varphi\,(\eta\,,\ \kappa\,,\ f\,,\ g,\ \Lambda) \tag{21}$$

The system is solved by finite differences method using a tridiagonal matrix algorithm. Values of the functions φ, Φ, and $\bar{h}$ are calculated at discrete points for each calculating layer of the calculating integration grid. The number of discrete points is $M = N = 401$. The system (19) is numerically solved using a program written in FORTRAN.

The equations are solved by iterative procedure. The usual values for the characteristic functions B, Q, and F_{mp} at a zero iteration are: $B^0_{K+1} = 0.469$, $Q^0_{1,K+1} = 1$, and $F^0_{\text{mp},K+1} = 0.4411$. The step for the variables is $\Delta\eta = 0.05$ and $\Delta f = 0.001$.

Results

The system (19) is numerically solved for each cross-section of the boundary layer, starting from the cross-section $f_1 = 0.00$. Solutions are obtained in tabular form.

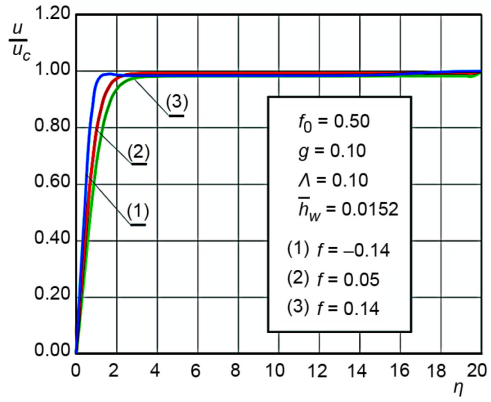

Figure 2. Distribution of the non-dimensional velocity

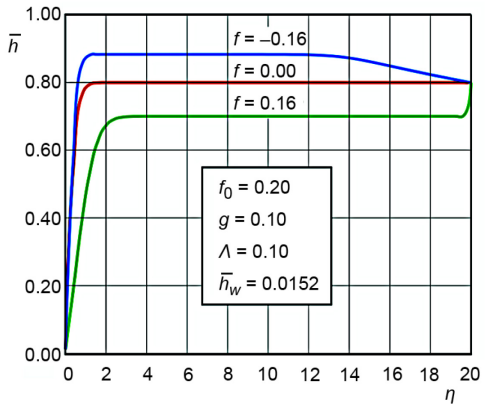

Figure 3. Distribution of the non-dimensional enthalpy

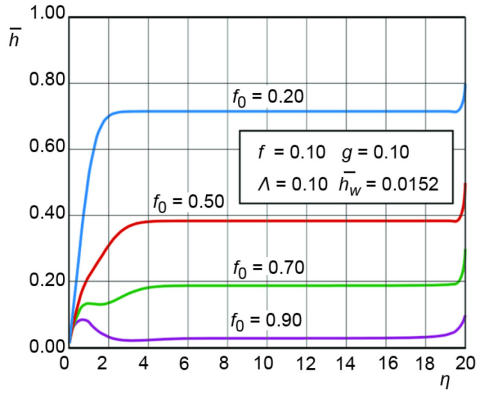

Figure 4. Distribution of non-dimensional enthalpy for different values of the compressibility parameter

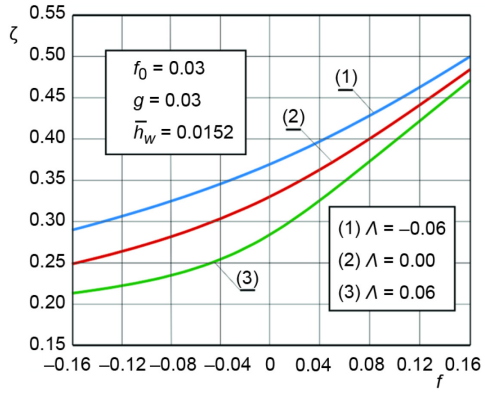

Figure 5. Distribution of the non-dimensional friction function for different values of the porosity parameter

Only some of the obtained results are given here in the form of diagrams. Figure 2 shows the diagram of the non-dimensional velocity $u/u_e = \partial\Phi/\partial\eta$ for three cross-sections of the boundary layer (f = -0.14; 0.05; 0.14) when $\kappa = f_0 = 0.50$.

The diagram in fig. 3 presents distribution of the non-dimensional enthalpy $\bar{h}$ for three cross-sections of the boundary layer. Figure 4 gives distribution of the non-dimensional enthalpy $\bar{h}$ at a cross-section of the boundary layer ($f = f_1 = 0.10$) for four different values of the compressibility parameter ($\kappa = f_0 = 0.20$; 0.50; 0.70; 0.90). Figure 5 shows a diagram of the non-dimensional friction function $\zeta(f)$ in the boundary layer for three different values of the porosity parameter Λ. Distribution of the non-dimensional friction function for different values of the magnetic parameter is given in fig. 6. Figure 7 shows distribution of the characteristic function F_{mp} for different values of the parameter Λ.

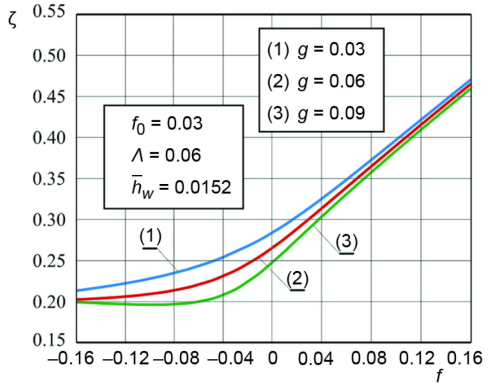

Figure 6. Distribution of the non-dimensional friction function for different values of the magnetic parameter

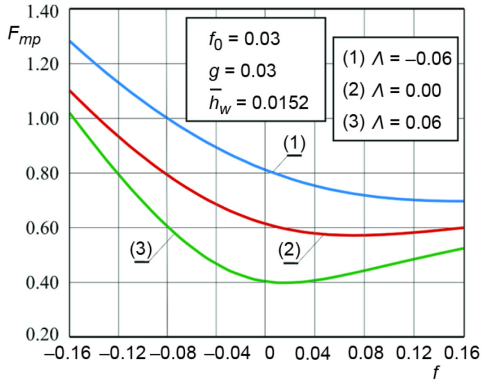

Figure 7. Distribution of the characteristic boundary layer function

Conclusions

This investigation has shown that Saljnikov's version of the general similarity method can be successfully used for solution of the studied problem of the axisymmetrical ionized gas flow in the boundary layer. Important quality results here obtained illustrate distributions of both physical and characteristic quantities at different cross-sections of the boundary layer.

Based on the diagrams here presented and others not shown, a general conclusion can be drawn that the distributions of the physical and characteristic quantities have the same behavior as with other problems of dissociated or ionized gas flow in the boundary layer.

The following conclusions can also be made.

- The non-dimensional flow velocity u/u_e (fig. 2) at certain cross-sections of the boundary layer on bodies of revolution converges very fast towards one, which is also characteristic for similar flow problems [25].
- The non-dimensional enthalpy $\bar{h}$ converges relatively fast towards the value at the outer edge of the boundary layer.
- Figure 4 shows that the compressibility parameter $\kappa = f_0$ has a significant influence on the distribution of the non-dimensional enthalpy, $\bar{h}$, in the boundary layer. As with other problems of the dissociated and ionized gas flow, this parameter changes the general behavior of the distribution of the enthalpy $\bar{h}$.

- The porosity parameter, Λ, has a significant influence on the non-dimensional friction function ζ (fig. 5). Consequently, it has a significant influence on the boundary layer separation point.
- The magnetic parameter, g, has a significant influence on ζ (fig. 6). By variation of the input parameters, assuming that the electroconductivity is the function of the longitudinal co-ordinate x, it has been concluded that the magnetic field will not postpone the separation of the boundary layer, as in [26-29].
- Behavior of the characteristic boundary layer function F_{mp} (fig. 7) is as expected and as usual [28].

In order to obtain more accurate results, the system (18) should be solved in a four-parametric approximation but without localization per the compressibility parameter, but this is fraught with difficulties, mainly of numerical nature.

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Nomenclature

B – boundary layer characteristic, [–]
B_m – induction of outer magnetic field [$= B_m(x)$], [Vsm^{-2}]
a, b – constants, [–]
c_p – specific heat of ionized gas at constant pressure, [Jkg^{-1}K^{-1}]
F_{mp} – characteristic boundary layer function, [–]
f_1 – first form parameter (= f), [–]
f_k – set of form parameters, [–]
g_1 – first magnetic parameter (= g), [–]
g_k – set of magnetic parameters, [–]
H, H_1 – boundary layer characteristic, [–]
h – enthalpy, [Jkg^{-1}]
$\bar{h}$ – non-dimensional enthalpy, [–]
h_e – enthalpy at the outer edge of the boundary layer, [Jkg^{-1}]
h_w – enthalpy at the wall of the body within the fluid, [Jkg^{-1}]
h_1 – total enthalpy in the outer flow, [Jkg^{-1}]
j, k – numbers, [–]
L – constant length, [m]
M – discrete point, [–]
Pr – Prandtl number (= $\mu c_p/\lambda$), [–]
Q – non-dimensional function, [–]
r – radius of the body of revolution in the meridian plane, [m]
s – new longitudinal variable, [m]
u – longitudinal projection of velocity in the boundary layer, [ms^{-1}]
u_e – velocity at the boundary layer outer edge, [ms^{-1}]
V_w – conditional transversal velocity, [ms^{-1}]
v – transversal projection of velocity in the boundary layer, [ms^{-1}]
v_w – velocity of injection (or suction) of the fluid, [ms^{-1}]
x, y – longitudinal and transversal co-ordinate, [m]
Z^{**} – function, [s]
z – new transversal variable, [m]

Greek symbols

Δ^* – conditional displacement thickness, [m]
Δ^{**} – conditional momentum loss thickness, [m]
Δ_1^{**} – conditional thickness, [m]
ζ – non-dimensional friction function, [–]
η – non-dimensional transversal co-ordinate, [–]
κ – local compressibility parameter (= f_0), [–]
Λ_1 – first porosity parameter (= Λ), [–]
Λ_k – set of porosity parameters, [–]
λ – thermal conductivity coefficient, [Wm^{-1}K^{-1}]
μ – dynamic viscosity, [Pa·s]
μ_0 – known values of dynamic viscosity of the ionized gas, [Pa·s]
μ_w – given distributions of dynamic viscosity at the wall of the body within the fluid, [Pa·s]
ν_0 – kinematic viscosity at a concrete point of the boundary layer, [m^{2}s^{-1}]
ρ – density of ionized gas, [kgm^{-3}]
ρ_e – ionized gas density at the outer edge of the boundary layer, [kgm^{-3}]
ρ_0 – known values of density of the ionized gas, [kgm^{-3}]
ρ_w – given distributions of density at the wall of the body within the fluid, [kgm^{-3}]
σ – electroconductivity, [Nm^{3}V^{-2}s^{-1}]
Φ – non-dimensional stream function, [–]
ψ – stream function, [m^{2}s^{-1}]
$\bar{\psi}$ – new stream function, [m^{2}s^{-1}]

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